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ABSTRACT

A broad range of research efforts are currently examining the quality and nature of the mathematics instructional program in the early grades. The purpose of this paper was to report on the assessment of a teaching-learning program in grade one classrooms. The major goals of the study were to: (1) identify patterns and change in children's thinking and solution procedures related to pre place value and informal place value tasks; (2) to assess the effects of an enriched number program on children's understanding and competence with multi-digit numbers; (3) to assess the effect of the program on teachers' and preservice teacher mentors' pedagogical beliefs; (4) to study mentor effectiveness in generating "inquiry mathematics" and small-group cooperative learning; and (5) to examine evidence of shared pedagogical beliefs resulting from interactions among researchers, teachers, and mentors. Twelve preservice mathematics teachers served as mentors for 41 first-grade students in 2 first grade classrooms of a University laboratory school for 8 weeks during each of 2 semesters. Children typically worked in pairs with one of the mentors with occasional whole- and small-group lessons. One-to-one assessment data of the children and observational, survey, and interview data of the teacher mentors were collected. Analysis of the data led to the following findings: (1) on common assessment items, average response rates increased from 65% to 78% by mid-year; (2) students adopted more flexible approaches to solving numerical problems; (3) teachers reported student success on some tasks that they would not normally expect them to succeed on at this stage; and (4) mentors revealed a more sophisticated approach to teaching both in their beliefs and in their actual practices. Assessment protocols and the instrument for mentor perceptions and beliefs are appended. (Contains 16 references.) (MDH)

First Grade Children's Understanding of Multi-Digit Numbers

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First Grade Children's Understanding of Multi-Digit Numbers

A major area of interest in this study is the improvement of early number experiences. It is clear that many young children have difficulty learning place-value concepts and skills, and that the quality and nature of the instructional program in the early grades merits close attention (e.g., Baroody, 1990; Fuson, 1991, 1990; Hiebert, 1991; Hiebert & Wearne, 1992; Kamii, 1991; Ross, 1991, 1989; Wearne, 1991).

Several researchers, including those just cited, currently are employing a broad range and variety of research efforts which target this problem. Fundamental philosophical differences characterize these studies.

Kamii (e.g., 1991) focuses on the internal construction of number meanings and does not use structured materials to support instruction or learning. Fuson (e.g., 1991, 1990) uses structured materials and emphasizes the connection between what is done (with manipulatives), said and written. She also emphasizes embedding place value learning in multidigit addition and subtraction computation work.

This instruction, however, is not presented in problem solving contexts as Carpenter & Fennema (1991) or Hiebert & Wearne (1992) might. The latter researchers balance their use of oral problem solving settings for addition and subtraction with an emphasis on making appropriate connections between what is said, written, and manipulated to nurture number sense and place value understanding. They have confined their efforts to the time span normally devoted to teaching place value and on related work in the typical first grade curriculum.

A second area of interest to this study concerns the nature of teachers (mentors) and children's roles as they do and talk about mathematics (Cobb,

Wood, Yackel, Nichols, Wheatley, Trigatti, & Perlwitz, 1991). It seems that in all instructional settings there is a power imbalance between the teacher and the student (Bishop, 1985), even when it is granted that the mentor's role in initiating and guiding mathematical negotiations is of prime importance and highly complex.

These roles impact on the development of group norms that are crucial to the constitution of an inquiry mathematics mode. For example, the establishment of social norms that enable children to engage productively in small group settings is essential to the success of a collaborative approach in the learning of mathematics.

The literature (Cobb, Wood, Yackel, Nichols, Wheatley, Trigatti, & Perlwitz, 1991) also suggests that teachers' (mentors') learning is an on-going, long-term process. Further, their classroom experiences are a critical source of pedagogical problems, the resolution of which involves the reorganization of their knowledge and beliefs about teaching and learning.

The purpose of this paper is to report on the assessment of a teaching-learning program in grade one classrooms. A major focus has been to identify patterns and change in children's thinking and solution procedures related to: a) pre place value problem tasks; and b) informal place value problem tasks as a result of a year-long, specially designed instructional program involving twelve preservice teacher mentors and two classes of first grade children.

In particular, the major goals of the study were:

- To assess the effects of an enriched number program on children's understanding and competence with multi-digit numbers.
- To assess the effect of the program on teachers' and mentors' pedagogical beliefs.

- To study mentor effectiveness in generating "inquiry mathematics" and small-group cooperative learning.
- To examine evidence of shared pedagogical beliefs resulting from interactions between researchers, teachers and mentors.

Theoretical Framework.

The theoretical framework for this research is drawn from aspects of Vygotsky's social learning theory (Vygotsky, 1978) and from Piaget's constructivist epistemology (Piaget, 1970), and emphasizes (1) interactive social settings; (2) problem solving; and (3) learning within a child's "zone of proximal development."

Vygotsky emphasizes the importance of social interaction in the learning process. Like Piaget, Vygotsky views learners as active organizers of their experiences but, in contrast, he emphasizes the fact that learning occurs twice, on two planes (Figure 1). First it occurs on the social plane, and then on the individual's psychological plane (Vygotsky, 1978, p. 57).

Social interaction in the instructional setting constitutes a critical source of opportunities to learn mathematics, posing situations and stimulating personal responses related to cognitive conflict, reflective abstraction and conceptual reorganization (Piaget, 1970) in mathematical learning.

Insert Figure 1 about here

This study is based on a constructivist approach to the development of number sense and place value understanding. Learning experiences and tasks are presented through group games and problem situations. When appropriate, children are involved in using physical materials to validate or aid

thinking. An on-going emphasis is highlighting the connection between what is done, said, and (eventually) written.

A further point is the fact that just as children are members of a classroom community, so are mentors and researchers. As a social community, mentors and researchers discuss knowledge and beliefs related to teaching and learning mathematics and develop alternative pedagogical approaches.

METHOD

Sample

The sample for the study was 41 first grade students in two first grade classrooms of a University laboratory school. Twelve students participating in mathematics education programs at the University served as teacher mentors and were also the sample for studying beliefs and preservice teacher ability to generate "inquiry mathematics" and small group cooperative learning.

Instructional Program

This study uses an "immersion" approach to address the problem of developing number sense and understanding of multi-digit numbers. Instruction occurred during 8 weeks of each semester, 20 minutes a day, 3 days a week in each of two classrooms. Approximately six whole class lessons conducted by one of the mentors and occasional lessons involving groups of four were carried out each semester.

Children typically worked in teams of two with a preservice mentor on problems related to the development of some aspect of number or place value. A set of 24 researcher-developed lessons each semester constituted the instructional program. Pre place value activities in first semester preceded the informal place value focus of second semester. The content of the fall and

spring programs are outlined in Figures 2 and 3. In fall, emphasis was given to pre-place value problem-solving tasks. In spring, informal place value problem-solving activities were highlighted.

Insert Figures 2 & 3 about here

Data Sources and Assessment

Both quantitative and qualitative data were gathered on the children and the teacher mentors. Regularly throughout each semester additional interview data was collected on four selected students and on the two classroom teachers.

A basic exploratory design was used to investigate change in children's understanding of multi-digit numbers over time. One-on-one assessment baseline data was obtained in the fall at the beginning of the study and at the end of each semester for all the first grade students. The researcher-constructed assessment protocols contained a number of common items as well as a number of new items on each successive administration. These instruments are included in Appendix A.

Non-structured participant observations were logged on each lesson by the mentors in conjunction with the teaching program, and non-participant observations were carried out by the researcher each session. In addition, one-on-one assessment interviews were administered by the mentors at the end of each semester.

Observational, survey and interview data were also gathered on each teacher mentor. A beliefs survey related to the teaching of mathematics was developed by the researchers and administered mid-year (see Appendix B).

Subsequently each teacher mentor was interviewed to gain further insights on their responses to the belief survey.

RESULTS

<u>Performance on Common Test Items</u>

Tables 1 and 2 present the means and standard deviations for the September and November assessments. An analysis is provided for both the common items and for the new items that were introduced in the November testing. Table 3 correlates the performance on the common test items for the September and November testing.

Insert Tables 1, 2 & 3 about here

In respect of the common items, the mean increase of approximately 1.6 in a total score of 9 is substantial for a group of 41 children. Moreover, the lower standard deviation for the November testing indicates that the children's scores are more closely clustered around the mean. That is, there is a more "even" performance in the November testing when compared to the September testing.

As indicated in Table 2, the maximum score possible for the total test was 17, and for the new items 18. Hence a mean of 13.32 is indicative of a correct response rate of more than 78%. The majority of the children scored between 10 and 16, and even on the 8 new items the group averaged a correct response rate of more than 73%.

The correlation coefficient indicates that while the children's performance at the two testing times is strongly related, only 29% of the variation in the November performance is explained by the children's initial level of

performance. The April testing, not yet complete, will include the common items along with additional items.

Error Analysis

Table 4 presents the level of performance of children's responses for both the September and November assessment protocols. For each item, the percentage of students responding correctly is given, which enables comparisons to be made between the item difficulties at each assessment time.

Insert Table 4 about here.

Consistent with the overall gain in mean performance, there is a steady gain in performance for each of the 9 common items. Of particular interest is item 6 which required the children to build 33 bears from 23. There was a 50% improvement in the use of grouping or building- on strategies. On item 7/10, the November performance was more than twice as good as the September performance on a task that required children to determine "what's missing" when two 10-trains and 1 unit were presented and 34 were required. This is a complex task for children at this age, whether they count on by "10s and 1s" or even by "1s."

In a similar way the tasks required in the new items involved quite extensive grouping and early numeration strategies, and these children were found to be already performing at above the 60% level on such items. Counting by 10s is strongly exhibited, and the children appeared to be especially adept at identifying numbers that were "a little more" and "a lot more" than 42. The latter result is somewhat surprising in view of the fact that the Kamii studies (1990) suggested that children great difficulties in identifying numbers "a lot more" than a given number.

Observations and Case Studies of Students and Teachers

Two students in each class were used to gain additional insights and to effect triangulation. Based on these observations and case studies, the researchers felt confident that students were learning viable and flexible strategies for attacking numerical problems. In fact there were instances of very creative approaches to quite complex problems for this age level. Two examples are presented in Figure 4.

Insert Figures 4 & 5 about here.

Both teachers were observed and interviewed regularly. They both agreed that the instructional program had a significant effect on their own teaching, and that they regularly followed up on lessons presented as part of the study. Helen noted that the project had benefited her as a teacher because it had helped her to zero in on the children's thinking as she observed their work and interactions with the mentors.

The teachers also commented that children often pursued ideas from a lesson until they felt personally satisfied with their own solutions or thinking in relation to a problem or an extension of a problem. On several occasions children completed problems that were not finished during a mentor lesson. One example is presented in Figure 5. The child's work is in response to the Pizza Party problem, which involved children solving a series of problems, the last of which was to determine how much money each person could spend for ice cream at Baskin-Robbins.

Mentor Beliefs.

The mentors expressed beliefs compatible with constructivist theories about learning and teaching (Cobb, Wood, & Yackel, 1990). On a nineteen-item subscale designed to measure agreement with statements about constructivist viewpoints, with one being strongly agree and five strongly

disagree, the mentors' mean score was 1.8 ($s = .34$). This places them on the scale between agree and strongly agree.

The mentors' responses on the questionnaire were supported by their actions in the classroom. Not once was a mentor observed telling a student the answer or the solution method they should use. Instead, questions such as "How did you get that?", "Why do you think that works?", and "Can you do it another way?" were commonly heard.

A sample of this behavior is illustrated in two mentor responses to the following question: "Have you been in a situation where children have struggled to solve a math problem?"

Sample interview response 1 (Rebecca): "Meridith had problems grasping what the problem was asking for. I tried to think of a creative way to explain what was meant. My instinct is to tell them what to do but I don't. I try to think of a hint or a way to make them think."

Sample interview response 2 (Tami): "Yes, it happens all the time. With the Toy Factory problem they had a hard time. I say things like, 'How could we figure this out?' 'What could we use to help us.' When they come up with an idea like using the cubes, I ask them, 'How would that help you?'"

Cooperative Learning

Although there were some instances of cooperative learning where students led each other to an improved understanding of a mathematical concept, the norm was less exciting. Typically the main cooperation was student- to-mentor rather than student-to-student. The presence of a mentor with each student team changed the group dynamics and may have limited cooperation between the students. Another explanation is that the groups were never required to turn in an assignment as a team; collected written work always came from individual students.

DISCUSSION

The study is still in progress, particularly in relation to the longitudinal performance of the students and, indeed, in relation to the effectiveness of the mentor program in generating inquiry mathematics and small group cooperative learning. At this point in time one needs to be cautious about overgeneralizing conclusions that derive from the study. Within these limitations the following findings are presented:

- On common assessment items, students went from an average response rate of 65% per item to 78% per item by mid-year.
- Observation indicates that students adopt more flexible approaches to solving numerical problems involving tasks like grouping, ordering, estimation and mental math.
- Teachers report student success on some tasks that they would not normally expect them to succeed on at this stage (e.g., adding in the hundreds; finding 9 groups of 30).
- Mentors reveal a more sophisticated approach to teaching both in their beliefs and in their actual practice (e.g., less directive, more constructivist in orientation; more able to engage children in problem solving than our experience would suggest for preservice teachers).

The strength of these preliminary findings provides impetus for the continuing study of the performance of this same group of children on problem-solving tasks involving multi-digit numbers. It also invites further investigation of mentors as teachers. In particular there is a need for continued study of the effect of mentor programs on beliefs about mathematics teaching, as well as the effectiveness of such programs in generating inquiry mathematics and facilitating small group collaborative learning. There is also a critical need to examine the use of the instructional program in a typical classroom setting.

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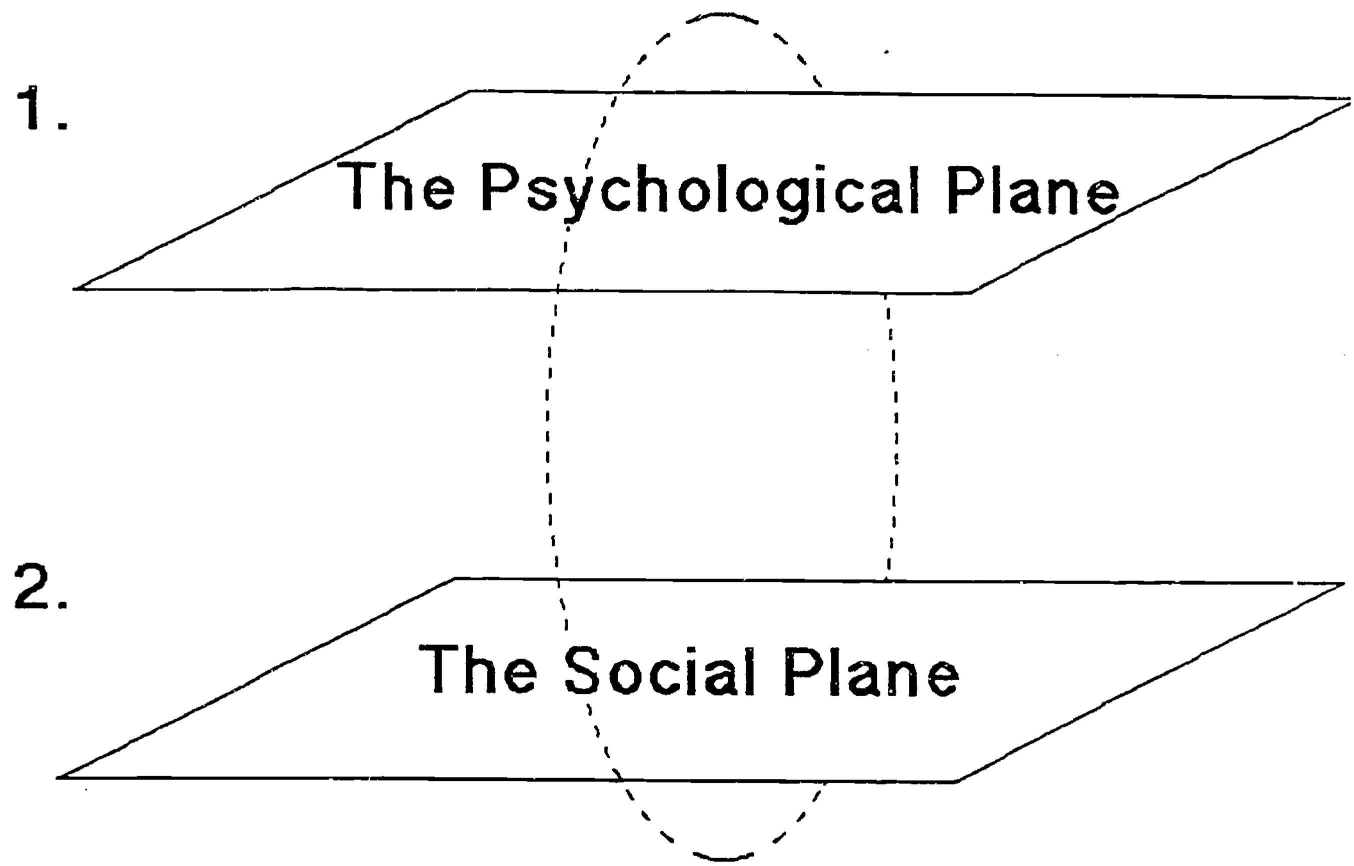


Figure 1. Two Levels of Learning

- **Estimating and counting (on-going!)**
- **Part-whole relationships**

  --Set partitioning (e.g. ways to make 5; 8; 10)

  --grouping (e.g. more/less than 5; 10)

- **Number relationships and ordering**
- **Number situations; +/- oral problems**
- **Informal grouping by tens (cubes in bag)**

Figure 2. Fall Program: Pre Place Value Problem Solving Activities

- **Part-whole relationships (10; 100; 50...)**

- **Estimation and general grouping activities**

- **Sequential grouping activities**

- **Equivalent groupings**

- **Counting and patterning (100 Chart)**

- **Situational problems: +/-, multi-digit numbers.**

Figure 3. Spring Program: Informal Place Value Problem Solving Activities

Interaction 1

The class topic was measurement. The question arose, "About how many jumbo clips will fit across the classroom?"

Nels said, "It would take too long to measure 1 by 1. Put chains of 30 clips end to end.. . . The children found that 9 groups of 30 were needed.

Different solutions for the total were found.

Jay: "Three 30s are 90; 90 + 90=180.

Cindy: "30 + 30 = 60; 60 + 30 = 90."..and so on to 180.

Tim: (Used unifix 10-trains and counted by 10s: 10, 20, ... to 180.)

Interaction 2

Zach: "I can figure out what 54 + 54 is. It's 108."

Mentor: "How did you figure that out, Zach?"

Zach: "50 + 50, taking away those 4s equals 100; and 4 + 4 = 8. So it's 108."

Figure 4. Classroom interactions resulting from the instructional program.

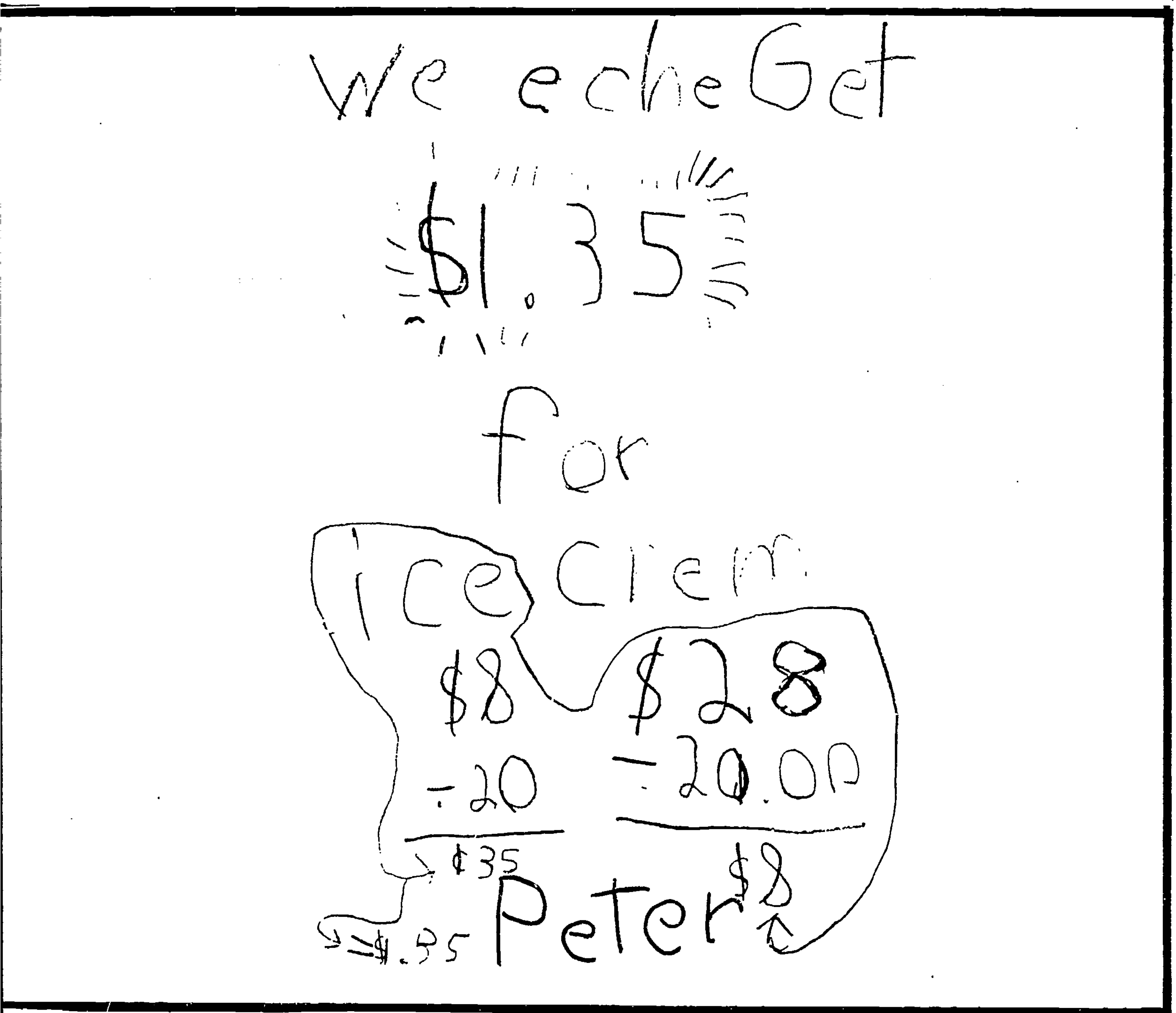


Figure 5. The Pizza Party Problem

TABLE 1 PERFORMANCE ON COMMON TEST ITEMS - SEPTEMBER AND NOVEMBER*

Time of Testing	Mean†	Standard Deviation
SEPTEMBER	5.9	1.75
NOVEMBER	7.475	1.15

†Total of 9 items

TABLE 2 NOVEMBER PERFORMANCE: TOTAL TEST AND NEW ITEMS

Test/Part	Mean††	Standard Deviation
TOTAL	13.325	2.87
NEW ITEMS	5.875	2.03

†† Total of 17 items including 8 new items

TABLE 3 CORRELATION OF PERFORMANCE ON COMMON TEST I ITEMS-SEPTEMBER AND NOVEMBER

r = O.54

• n - 42 students

TABLE 4 ERROR ANALYSIS -PERCENT CORRECT: SEPTEMBER / NOVEMBER

	ITEM #		SEPTEMBER	NOVEMBER
COMMON ITEMS	1	(How many?)	78%	93%
	2	(Conservation)	48%	58%
	3a	(1-digit recognition)	95%	100%
	3b	(Show the number)	88%	98%
	4	(Count on from 9	83%	95%
		objects)	75%	93%
	5a	(2-digit recognition)	73%	78%
	5b	(Show the number)	28%	78%
	6	(Show 10 more)	20%	45%
	7/10	(Tell what's missing)		
NEW ITEMS	7	(How many? (2 digits))		75%
	8	(How many hidden?(10))		68%
	9	(How many hidden?(12))		60%
	11	(Sequential counting by		73%
		10s and 1s)		100%
	12a	(2- digit recognition: 42)		93%
	12b	(a little more)		88%
	12c	(a lot more)		

Appendix A

Assessment Protocols

Assessment - September 18, 1991

(DO)	(SAY)
1. Put out 18 bears (random placement):	"How many bears are there?"
2. Line the bears up side by side:	"What is this number?"
(Clear the bears away)	
3. Write the number **9** on a piece of paper:	"What is this number?" "Put out this many bears."
(If they can't read it, tell them.)	
4. Put out 2 more bears:	"How many now?"
(Clear the bears away.)	
5. Write the number **23** on a piece of paper:	"What is this number?" "Put out this many bears."
6. Write the number **33**:	"Now show me 33 bears."

7. Write the number **34** on a piece of paper. Put out 2 ten trains and 1 extra cube.

"There should be 34 here. What's missing."

8. **"I'm going to count by tens: 10, 20, 30." Can you count from the start like I did and just keep going? Try it. 10, ..."**

Instructor________________________________ Sept. 1991

Name of child ____________________
Class________________

1. Check one: Correct________ Not correct__________

   Describe any system used in counting. (by 1s, by 10s, touched and moved the bears, etc...)

2. 1 point if they say 18 right away without counting.________
   0 points otherwise _______
3. 1 point if they can read the number. ___________
   0 points otherwise _______

   1 point if they can put out nine objects _________
   0 points otherwise. _______

4. 1 point if they can count on from 9. ____________
   0 points otherwise. _______
5. 1 point if they can read the number. ___________
   0 points otherwise. _______

   1 point if they can put out that many bears. __________
   0 points otherwise. _______
   1 extra point if they put out 2 groups of 10 and 3. _________
6. 1 point if they count only the 10 extra bears. __________
   0 points otherwise. _______
7. 1 point if they can tell there are 1 ten, 3 extra hidden ________
   0 points otherwise. _______

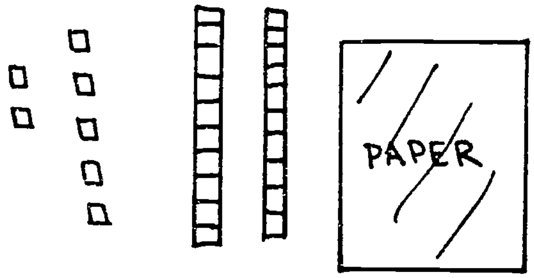
8. 1 point if they can count by 10s. _________
   0 points otherwise. _______

Assessment - November 25, 1991

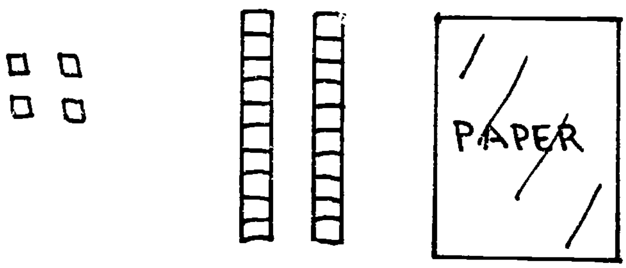
Materials: Bears (35); Pencils, Paper, Bag of unifix cubes

(DO)	(SAY)
1. Put out 17 bears: (random placement)	"How many bears are there?
2. Line the bears up side by side:	"How many bears now?" (Clear the bears away when finished)
3. Write the number 9 on piece of paper:	"What is this number?" (If they can't read it, tell them) "Put out this many bears."
4. Put out 2 more bears:	"How many now?" (Clear the bears away)
5. Write the number 23 on a piece of paper:	"What is this number?" "Put out this many bears." (Clear the bears away when finished)
6. Write the number 33:	"Now show me 33 bears."
7. Put out 3 ten trains and 1 extra.	"How many cubes are there altogether?"

(DO)	(SAY)
8. Put out 27 as shown	"There are 10 hidden under the paper." "How many are there altogether?"



(DO)	(SAY)
9. Put out 24 as shown.	"There are 12 hidden under the paper." "How many are there altogether?"

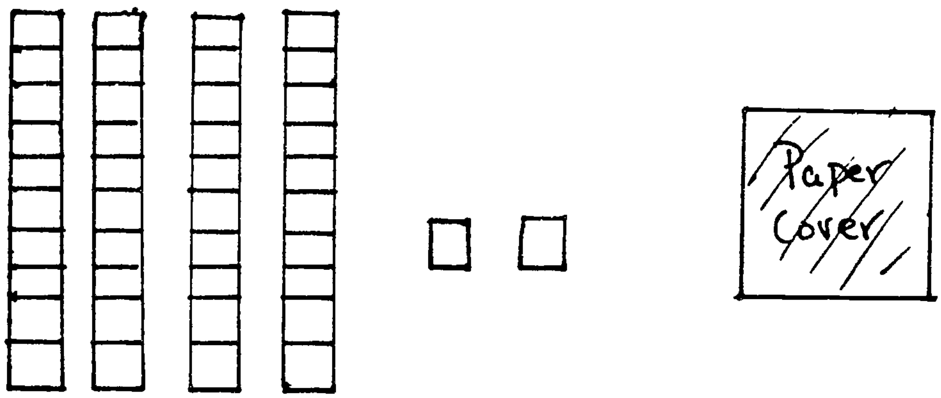


(DO)	(SAY)
10. Write the number 34 on piece of paper. Put out 2 ten trains and 1 extra cube.	"There should be 34 here," "What's missing?"

11. [uncovering task]
Gradually uncover the cubes, as suggested by the arrows.

Pause at each arrow and ask:

"How many are there now?"



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12. Write 42 on a piece of paper.

"What number is this?"

"Write a number that's a little more than 42. ________

"Now write a number that's a lot more than 42. ________

Instructor_______________________ Date:________

Name of child ____________________

Class_________________________

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1. Check one: Correct______ Not correct________

   Describe any system used in counting. (by 1s, by 10s, touched and moved bears, etc...)

2. 1 point if they say 17 right away without counting. ______
   0 points otherwise.______

3. 1 point if they can read the number. _______
   0 points otherwise. ________

   1 point if they can put out 9 objects. ________
   0 points otherwise. ________

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4. 1 point if they can give correct answer immediately or count on from 9. _______
   0 points otherwise. ________

5. 1 point if they can read the number._______
   0 points otherwise. ________

   1 point if they can put out (23) bears. ________
   0 points otherwise. ________

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6. 3 points if they lay out 10 bears right away. ________
   2 points if they put out 3 groups of 10 and 3. _______
   1 point if they count 33 singles. ________
   0 points otherwise. ________

7. 1 point if they can tell there are 3 tens, 1 extra. _________
   0 points otherwise. ________

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8. 2 points if they can count by 10s and 1s to get 30 and then count in the extra 1s--to get 37.________
   1 point if they can count by 1s to get 37.______
   0 points otherwise. ________

9. 2 points if they count by 10s and 1s to obtain an answer (36). _________
   1 point if they can count by 1s to obtain an answer (36). ________
   0 points otherwise. ________

10. 2 points if they count by 10s and 1s to determine that there is 1 ten and 2 ones missing.________
1 point if they have to count by 1s to determine if there is 1 ten and 2 ones missing. _______
0 points otherwise. _______

11. Record the child's oral count.

1 point if the count is 10, 20, 30, 40, 41, 42 (some children might say 40, 42. That is OK.)______
0 points otherwise. ________

12. 1 point if child reads 42. _______
0 otherwise. _______

Record the number given for "a little more than 42" _________

Record the number given for " a lot more than 42"_________

Assessment - April, 1992

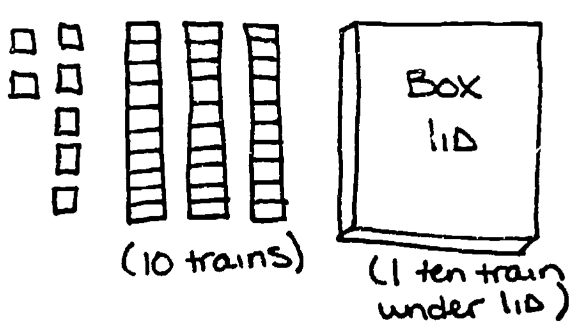
Materials: Bag of 100 cubes, box lid (#6, #7), 24 +21 and 50 40 30 20 60 for #9, 37 +17 and 50 40 30 70 60 for #12, hundreds chart and peep hole card, scrap paper and pencil.

(DO)	(SAY)
1. Put out 19 cubes (random placement)	"How many cubes and there?"
2. Line the cubes up side by side.	"How many cubes now?" (Clear the cubes away when finished)

(Have at least 4 ten trains and 10 loose cubes available)

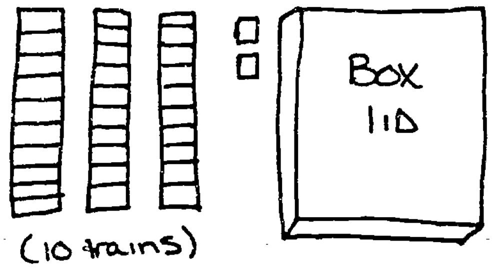
3. Write the number 25 on a piece of paper.	"What is this number?" "Put out this many cubes."
4. Write the number 35.	"Now show me this many cubes."

5. Put out 6 ten trains and 4 extra.	"How many cubes are there all together?"
6. Put out 37 cubes as shown.	"There are 10 hidden under the box." "How many are there all together?"



(DO) (SAY)

7. Put out 32 cubes as shown.



(10 trains)

"There are 13 hidden under the box."

"How many are there all together?"

8. Write the number 46 on a piece of paper.
Put out 3 ten trains and 4 extra cubes.

"There should be this many (point to ~~43~~ 46)."

"How many are missing?"

9. Show the card

$$\begin{array}{r} 24 \\ +21 \end{array}$$

and

50		40
	30	
20		60

"Estimate to see whether the answer is in the 20s, 30s, 40s, 50s or 60s."

Circle your estimate."

10. Use #11, yellow sheet, attached.

11. Provide pencil and paper.

"Write 27."
"Now write a number that's a lot more than 27."

(DO)	(SAY)

12. Show the card.

37
+17

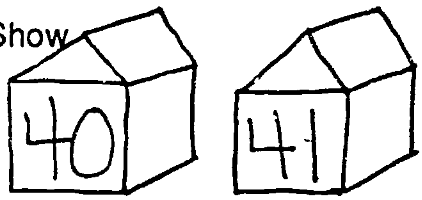
and

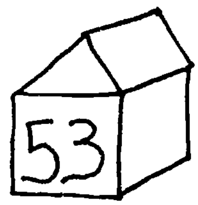
50 40
30
70 60

"Estimate to see whether the answer is in the 30s, 40s, 50s,60s or 70s."

"Circle your estimate."

13. Show





"A builder needs numbers from 40 to 53 for the houses on one block. He can only buy single digit numerals. "

"How many houses are on the block?"

"How many 4s does he need to buy?"

"How many digits will he have to buy two of?"

Instructor__________________ Date:__________

Name of child_________________

Class________________________

1. ___ Check if correct.

Describe any system used in counting (by 1s, by 10s, touched and moved cubes, etc...).

2. ___ 1 point if they say 19 right away without counting.
___ 0 points otherwise.

3. ___ 1 point if they can read the number.
___ 0 points otherwise.

___ 1 point if they can put out 25 cubes.
___ 0 points otherwise.

4. ___ 2 points if they can give correct answer immediately (put out a ten-train).
___ 1 point if they count on from 25.
___ 0 points otherwise.

5. ___ 2 points if they count ten-trains and ones to get 64.
___ 1 point if they count on from some subset of ten-trains, (e.g. 30, 40, 50, 60, 61, 62, 63, 64).
___ 0 points otherwise.

6. ___ 2 points if they can count by 10s and 1s to get 40 and then count on 10 more to get 47 (i.e. 37, 47).
___ 1 point if they can count on by 1s from 37 to get 47.
___ 0 points otherwise.

7. ___ 2 points if they count by 10s and 1s to get 32, then count on (42, 43, 44, 45) to obtain an answer (45).
___ 1 point if they can count on from 32 to get answer (45).
___ 0 points otherwise.

8. ___ 3 points if they can count by 10s and 1s to get 34 and then think 10s and 1s to get 46. They solve the problem correctly, (one 10 and two1s).

___ 2 points if they think correctly but get wrong answer.

___ 1 point if they have to count on from 34 to determine that there is 1 ten and 2 ones missing.

___ 0 points otherwise.

---

9. ___ 1 point if estimate is reasonable and children do not count on by ones.

___ 0 points otherwise.

---

10. ___ Record the child's oral count.

___ 1 point if the count is 10, 20, 30, 40, 41, 42 (some children might say 40, 42. That is OK.)

___ 0 points otherwise.

---

11. ___ 1 point if they write "27" correctly (accept reversals).

___ 0 points otherwise.

___ Record the number given for "a lot more than 27".

---

12. ___ 1 point if estimate is reasonable and children do not count on by ones.

___ 0 points otherwise.

---

13. Outline the child's solution strategy.

___ Record number of houses child suggest.

___ 1 point if correct.

___ 0 points otherwise.

___ Record number of 4s child suggests.

___ 1 point if correct.

___ 0 points otherwise.

___ Record number child suggests.

___ 1 point if correct.

___ 0 points otherwise.

Appendix B

Mentor Perceptions and Beliefs

Social Security #:______________

BELIEFS ABOUT MATHEMATICS TEACHING

Express the extent of your agreement with each of the following statements by marking the appropriate response.

(SA) strongly agree
(A) agree
(U) undecided
(D) disagree
(SD) strongly disagree

* 1. A key responsibility of a teacher is to encourage children to explore their own mathematical ideas.
(SA) (A) (U) (D) (SD)

* 2. Ignoring the mathematical ideas that children generate themselves can seriously limit their learning.
(SA) (A) (U) (D) (SD)

3. Knowing how to solve a mathematics problem is as important as getting the correct solution.
(SA) (A) (U) (D) (SD)

* 4. Acknowledging multiple ways of mathematical thinking is inefficient and may confuse children.
(SA) (A) (U) (D) (SD)

5. A vital task for the teacher is motivating children to resolve their own mathematical problems.
(SA) (A) (U) (D) (SD)

6. Teachers can create, for all children, a non-threatening environment for learning mathematics.
(SA) (A) (U) (D) (SD)

7. I would feel uncomfortable if a child suggested a solution to a mathematical problem that I hadn't thought of previously.
(SA) (A) (U) (D) (SD)

* 8. Teachers of mathematics should be fascinated with how children think and be intrigued by alternative ideas.
(SA) (A) (U) (D) (SD)

9. Learning mathematics involves a lot of memorizing.
(SA) (A) (U) (D) (SD)

10. A teacher's main function is to provide children with solutions to their mathematical problems.
(SA) (A) (U) (D) (SD)

11. Listening carefully to the teacher explain a mathematics lesson is the most effective way to learn mathematics.
(SA) (A) (U) (D) (SD)

* 12. Teachers must be able to represent mathematical ideas in a variety of ways.
(SA) (A) (U) (D) (SD)

13. Mathematics is best learned individually.
(SA) (A) (U) (D) (SD)

14. Although there are some connections between different areas, mathematics is mostly made up of unrelated topics.
(SA) (A) (U) (D) (SD)

* 15. Persistent questioning has a significant effect on children's mathematical learning.
(SA) (A) (U) (D) (SD)

16. Teachers always need to hear children's mathematical explanations before correcting their errors.
(SA) (A) (U) (D) (SD)

* 17. Effective mathematics teachers enjoy learning and "doing" mathematics themselves.
(SA) (A) (U) (D) (SD)

18. If a child's explanation of a mathematical solution doesn't make sense to the teacher it is best to ignore it.
(SA) (A) (U) (D) (SD)

* 19. It is important to cover all the topics in the mathematics curriculum in the textbook sequence.
(SA) (A) (U) (D) (SD)

* 20. Providing children with interesting problems to investigate in small groups is an effective way to teach mathematics.
(SA) (A) (U) (D) (SD)

21. As a result of my experiences in mathematics classes, I have developed an attitude of inquiry.
(SA) (A) (U) (D) (SD)

* 22. It is not necessary for teachers to understand the source of children's errors; follow-up instruction will correct their difficulties.
(SA) (A) (U) (D) (SD)

* 23. Telling children the answer is an efficient way of facilitating their mathematics learning.
(SA) (A) (U) (D) (SD)

24. It is the teacher's responsibility to provide the children with clear and concise solution methods for mathematical problems.
(SA) (A) (U) (D) (SD)

25. In mathematics, problems can be solved without using rules.
(SA) (A) (U) (D) (SD)

26. It is important for children to be given opportunities to reflect on and evaluate their own mathematical understanding.
(SA) (A) (U) (D) (SD)

* 27. There is an established amount of mathematics content that should be covered at each grade level.
(SA) (A) (U) (D) (SD)

28. Children can learn more mathematics working together than by themselves.
(SA) (A) (U) (D) (SD)

* 29. It is important for teachers to understand the structured way in which mathematics concepts and skills relate to each other.
(SA) (A) (U) (D) (SD)

30. Allowing a child to struggle with a mathematical problem, even feel a little tension, can be necessary for learning to occur.
(SA) (A) (U) (D) (SD)

* 31. Mathematical material is best presented in an expository style: demonstrating, explaining, and describing concepts and skills.
(SA) (A) (U) (D) (SD)

32. Children always benefit by discussing their solutions to mathematical problems with each other.
(SA) (A) (U) (D) (SD)

33. My mathematics teachers would often show me different ways to solve a problem.
(SA) (A) (U) (D) (SD)

* 34. It is important that mathematics content be presented to children in the correct sequence.
(SA) (A) (U) (D) (SD)

35. Justifying the mathematical statements that a person makes is an extremely important part of mathematics.
(SA) (A) (U) (D) (SD)

*Kuhs, T. M., & Ball, D.L. (1986). *Approaches to Teaching Mathematics: Mapping the Domains of Knowledge, Skills, and Dispositions Implication for Studying the prepartation of Teachers.* Research memo (Office of Educational Research and Improvement/Department of Education).

#Rich, B. & Otto, A. (1990). Beliefs about Mathematics. (Survey prepared at Illinois State University).